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Describing Airspace Complexity: Airspace Response to Disturbances

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In ongoing efforts to balance air traffic demand and airspace capacity, airspace complexity stands as a fundamental research problem. This paper proposes a new notion of airspace complexity by capturing how difficult a given traffic situation is in terms of the control activity required to accommodate disturbances such as the entrance of another aircraft into the airspace. A complexity map is introduced that offers a graphical view of the control activity required to accept a new aircraft as a function of its essential parameters. The proposed method is illustrated with examples related to traffic flow management and dynamic airspace configuration.

Nomenclature

d	=	separation standard
$x_i(t)$, $y_i(t)$	=	position of aircraft i at time t
V	=	speed of aircraft
θ_i	=	initial heading angle of aircraft i
θ_{ni}	=	new heading angle of aircraft i after conflict resolution

I. Introduction

THE air traffic management (ATM) system provides services for safe and efficient aircraft operations to transport people and goods [1,2]. The airspace in the United States is divided into 22 centers, including Alaska and Hawaii, and each center is subdivided into sectors. Aircraft inside each sector are controlled by one or two sector controllers for which the primary task is assuring proper separations between aircraft. On the other hand, the traffic management coordinators focus on orderly and efficient traffic flows through multiple sectors to ensure efficient use of the National Airspace System (NAS). Increased demand on the air transportation system will degrade the quality of services in the near future, and much effort has been put into improving the current ATM system.

In the current system, each sector has a capacity limit that is defined by the maximum number of aircraft that can be handled in the sector during a given time period, which is called the monitor alert parameter (MAP) [3,4]. The MAP is primarily determined by the sector geometry such as the size of the sector, but it can also be affected by dynamic environmental factors such as convective weather. For example, in general, a larger sector has a bigger MAP than a smaller sector, but its MAP can significantly drop in a bad weather condition. If the number of aircraft inside a sector is expected to be more than its MAP, air traffic managers can respond by either restricting incoming traffic flows [i.e., traffic flow management (TFM)] or by increasing the capacity of the sector [i.e., dynamic airspace configuration (DAC)].

Airspace complexity is intimately tied to TFM, because airspace capacity, which drives traffic flow management policies, directly depends on its current and future traffic configurations [5–7]. For example, the concept of multisector planning proposes to resolve an overly demanding traffic situation well in advance by regulating traffic flows based on estimated airspace complexity [8]. Likewise, DAC examines how airspace managers may reconfigure the airspace to meet variations in air traffic demand. For example, they may allocate and deallocate the airspace in accordance with their ability to accurately forecast airspace complexity [9,10].

The common practice of counting aircraft in a given sector inadequately captures the complexity of a given air traffic situation. As a consequence, several new concepts have been proposed. One of them, dynamic density, attempts to identify various factors affecting controllers' workload and build a measure of airspace complexity by combining these factors. Numerous simulations have recorded measures of controller workload such as their self-reports of perceived workload or the number of communications as an indication of their workload. The weighting of the factors was determined through either linear or nonlinear regressions of the recorded data [1,11–15]. Taking another approach, task-based workload models have also been proposed [7,16,17]. Despite many attempts to relate airspace complexity to measures referenced to controllers' workload, this type of measures can be idiosyncratic and situation-specific [18,19]. Although airspace complexity accounts for a large portion of controller workload, controller workload is also related to other mediating factors such as an individual cognitive process [5,18–22]. Furthermore, previous research has pointed out that controller workload will not be properly understood until we understand how human controllers change their control strategies and regulate their cognitive workload [18–21].

Other research has focused on purely analytical airspace-complexity indicators based on geometric attributes of traffic without attempting to relate airspace complexity with controller workload [23–28]. Taking a different approach, metrics have been proposed reflecting the lack of structure of the traffic based on relative aircraft positions and velocities [23], the fractal dimensions of the traffic flows [24]. Another study mapped the airspace onto a simpler dynamic system to compute its Lyapunov exponents map [25,26]. Still another approach investigated airspace complexity by classifying groups of related aircraft as clusters [27,28]. The performance and stability of intersecting traffic flows have also been investigated [29,30].

Our intent is to build upon these more analytic works by providing an approach that focuses on airspace as a closed-loop controlled system responding to disturbances. Complexity of an airspace will be described by using the probe of adding a hypothetical aircraft as a disturbance. To be operationally useful, a method of describing airspace complexity should not only identify overly complex traffic situations, but should also suggest how an airspace manager can resolve those overly demanding traffic situations. Therefore, this paper focuses on providing more detailed information of how a given

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traffic situation responds to disturbances rather than providing a single scalar metric of airspace complexity.

In the first part of the paper, we explain this approach to describe airspace complexity. We then give numerical examples to demonstrate the method, and we show the usefulness of the proposed method in a manner relevant to applications of TFM and DAC. Finally, we discuss the results obtained and describe future work.

II. New Notion of Airspace Complexity

This paper will describe the complexity of a given sector with its instantaneous traffic situation. The airspace, together with the aircraft inside it, can be viewed as a closed-loop controlled system in which many events, without intervention, can create safety violations between aircraft, as shown in Fig. 1 [31]. Air traffic controllers currently provide the response to such events; these events may be handled with automation assistance.

Definition 1: A *disturbance* is any event that may require intervention to maintain the safe, orderly flow of aircraft.

Disturbances include new aircraft entering into the airspace, nonconforming aircraft inside the sector [32], and convective weather. It should be noted that convective weather can be considered as both a disturbance and an environmental factor. If we focus on how convective weather will disturb an airspace, it should be considered as a disturbance. However, it should be considered as an environmental factor if we focus more on managing the traffic flows in the presence of known convective weather.

From this controlled system perspective, the complexity of a traffic situation can be described in terms of how the airspace (i.e., traffic configuration and control architecture) reacts to disturbances. Furthermore, the response of the system to the disturbance represents the difficulty of dealing with traffic in the face of such disturbances, and it is captured by control activity defined in the following way.

Definition 2: *Control activity* is the degree of response required to resolve any conflict arising from a disturbance.

Control activity depends on the control strategy used to manage the traffic. As shown in the previous studies [29,30], a traffic situation can be extremely difficult to manage by some traffic control strategies, but it might not be difficult for others. Intrinsic airspace complexity might be described by computing the minimum control activity over all traffic control strategies available, but this paper describes airspace complexity for a given traffic control strategy.

Different types of control activity can be used for different analysis purposes. The flexibility of choosing control activity can provide broader insight into the airspace complexity. For example, although this paper focuses on a more analytical approach for airspace complexity, the results can be related to human controller workload by a proper choice of measure of control activity; numerous simulations can be performed to examine how a given measure of control activity represents controller workload by computing its correlation with controller workload indicators.

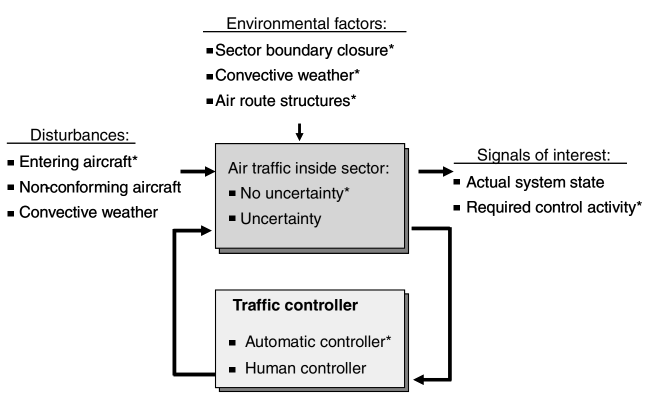


Fig. 1 Airspace as a closed-loop controlled system (* denotes factors that are specifically examined in this paper).

Now the complexity of a given traffic situation can be described using the control activities corresponding to a given set of disturbances as follows.

Definition 3: A *complexity map* details control activity as a function of the parameters describing the disturbances.

The difficulty of dealing with a given traffic situation depends on the disturbance. In other words, a traffic situation can suffer significantly from some disturbances but it might not be affected by others. A single scalar metric cannot properly describe the traffic complexity, and the complexity map proposed in this paper serves as a means for analyzing airspace response to a set of disturbances.

A partial order between complexity maps can be defined as follows.

Definition 4: One complexity map is less than the other if and only if control activity of one map is less than control activity of the other map over all disturbances.

In addition to describing the complexity of a given traffic scenario, it is also important to assess how airspace complexity is affected by environmental factors such as convective weather shutting down a region within the airspace or a partial closure of a sector boundary due to dynamic airspace management restrictions on traffic flow. Quantifying the degree of increased complexity caused by these environmental changes is necessary for TFM and DAC. As shown later, the method proposed in this paper will allow airspace managers to quickly apprehend detailed information on how airspace complexity is affected by these environmental changes.

III. Airspace Model

A. Aircraft and Sector Models

For simplicity, this paper considers only the horizontal motion of aircraft. The following 2-D kinematical model is used for each aircraft inside the sector:

$$\dot{x}_i(t) = V\cos\theta_i, \qquad \dot{y}_i(t) = V\sin\theta_i \tag{1}$$

where $1, 2, \ldots, N$.

Each pair of aircraft is forbidden to be closer than a given separation standard d. In other words, a safety violation between aircraft i and j occurs if

$$\sqrt{[x_i(t) - x_j(t)]^2 + [y_i(t) - y_j(t)]^2} < d \tag{2}$$

Definition 5: Two aircraft are in *conflict* if propagation of their positions in time, without intervention, creates a safety violation between them at some point in time.

Definition 6: A traffic situation is *conflict-free* if propagation of all aircraft position in time, without intervention, do not create any safety violation between aircraft at any time.

We can consider each aircraft as a disc with radius $d/2$; safe separation is ensured when these discs do not intersect each other. In this paper, all aircraft have the same constant velocity, and each aircraft is only allowed to change its heading angle to avoid conflicts with others.

The sector boundary is approximated here by a circle. The sector boundary should be explicitly defined because it defines when a disturbance, such as an additional aircraft, enters the system. The idea of using a predefined sector boundary is relevant to current operation of the NAS and has been used in previous research for mathematical analyses of air traffic control systems [29,30,33].

It should be noted that the proposed method can also be applied to describing local traffic complexity of groups of aircraft independently of geographic sector boundaries in the NAS. This type of complexity has been recently called gaggle density [34]. For example, in a free-flight environment, in which aircraft choose their own air routes, the ATM system may still need to prevent aircraft from entering any locally complex areas in which separation may be difficult to achieve without excessive control activity [34].

The method proposed in this paper does not preclude the use of more extensive models. For example, we may use a three-dimensional airspace model, and different speeds and altitude changes can be allowed for aircraft. Likewise, the method can accommodate different shapes for the sector boundary, as discussed later.

B. Control Architecture

The control architecture generates the system's reaction to a disturbance in the form of conflict avoidance maneuvers. There are several control architectures we may choose from, and the description of airspace complexity depends on which control architecture is chosen [19,20,29,30]. Our aim to build a tool for fast computer-based complexity evaluations requires the choice of a computer-based conflict resolution algorithm. These could consist of centralized or decentralized optimal conflict solvers or of rule-based solvers and could resolve conflicts simultaneously or sequentially [35]. The choice of solver might itself be driven by the goal of the analysis or by the control architecture used in real operations. For illustrative purposes, our work uses two different control architectures, which we now describe. Our purpose is *not* to describe these algorithms at the highest level of detail, but rather to illustrate how our approach can incorporate different conflict solvers without difficulty.

1. Centralized Conflict Resolution Algorithm Using Mixed-Integer Programming

The first control architecture is the centralized conflict solver proposed by Pallottino et al. [36]. The solver assumes that all aircraft have the same speed (although this assumption can be lifted), such that the available decision variables are only the aircraft headings. This traffic control architecture assumes that all conflicts are resolved simultaneously when a disturbance enters the system. This traffic control architecture does not distinguish between safety violations happening outside the sector from those inside the sector, although our method does not preclude this. Considering a set of N aircraft, we denote θ_i as the initial heading of aircraft i and θ_{ni} as its new heading obtained from conflict resolution. Among many possible solutions, Pallottino et al. chose the one minimizing

$$\sum_{i=1}^{i=N} |\theta_{ni} - \theta_i| \tag{3}$$

subject to all aircraft avoiding conflicts among themselves. This problem may be formulated as a mixed-integer program for which the solution can be handled by fast-optimization tools such as CPLEX [37].

2. Sequential Conflict Resolution Algorithm

Another conflict resolution algorithm that may stand closer to air traffic control by humans solves conflicts sequentially. In the previous conflict resolution algorithm, all conflicts are resolved simultaneously as soon as a disturbance enters the system. However, in real operation, air traffic controllers command only one aircraft at a time to change its heading. Then they command another aircraft to change its heading after a certain time interval. Therefore, this sequential conflict resolution algorithm resolves conflicts in the following way:

1) Create a sequence of aircraft (AC) inside the sector (e.g., AC1-AC2-AC3-AC4, etc.). In this paper, we assign a number to each aircraft from the westmost aircraft to the eastmost aircraft. If two aircraft are aligned vertically, we assign a number to the southmost aircraft. Other indexing rules are possible.

2) AC1 maintains its trajectory, whereas AC2 changes its trajectory to avoid a conflict with AC1.

3) After a certain time interval, AC3 changes its trajectory to avoid conflicts with AC1 and AC2, whereas AC1 and AC2 maintain their trajectories.

4) We proceed to each subsequent aircraft k and change its trajectory to avoid conflicts with the previous aircraft $1, 2, \ldots, k-1$. The previous aircraft maintain their trajectories as previously determined.

The solver also assumes that all aircraft have the same speed such that the available decision variables are only the aircraft headings and each heading change is instantaneous. This solver does not distinguish between safety violations happening outside the sector from those inside the sector.

C. Environmental Factors

We are often interested in analyzing how airspace complexity is affected by changes in environmental factors, and this paper considers adjacent sector closure and convective weather. For the adjacent sector closure, the traffic inside the sector is not allowed to exit through the part of the sector boundary in which the adjacent sector is closed. Therefore, the case of adjacent sector closure is modeled as a partial closure of the sector's boundary. The case of convective weather is modeled as a circular shutdown region in the sector, and the traffic inside the sector is not allowed to go through this part of airspace. More static environmental factors such as a given air-route structure are investigated in this paper by assessing their complexity over a wide range of traffic situations.

IV. Response of Airspace to Entering Aircraft

A. Details of the Disturbances

Among the many different types of disturbances, this paper investigates the response of the airspace to a specific type of disturbances (i.e., aircraft entering into the airspace). Any aircraft entering the sector at any heading and entry location will be considered. An entering aircraft is at the sector boundary when it appears, and it has the same speed as the other aircraft in the sector. A complexity map is computed by considering all possible entering aircraft spanning all entering points on the boundary and all headings. To define instances of entering aircraft, the following definitions are introduced and illustrated in Fig. 2.

Definition 7: The *entering-aircraft position angle* is the entry location of an aircraft into the sector, defined in angular coordinates from the north.

Definition 8: The *entering-aircraft bearing* is the relative track of an entering aircraft with respect to the radial line connecting the aircraft to the center of the sector. When the bearing is zero, the entering aircraft is moving straight toward the center of the sector. Figure 2 illustrates a positive bearing.

A complete set of disturbances encompasses all possible entering-aircraft position angles and bearings. For a circular sector boundary, the entering-aircraft position angle spans 0 to 360 deg. We consider only entering aircraft that will be inside the sector for some period of time (i.e., with bearing angles between −90 and 90 deg).

As mentioned before, the method can be extended to using a different shape of sector boundary. For example, for an arbitrary shape of the sector boundary, the entry location of an aircraft can be defined by the parameter s along the sector boundary, as shown in Fig. 3, and entering-aircraft bearing can be defined with respect to the

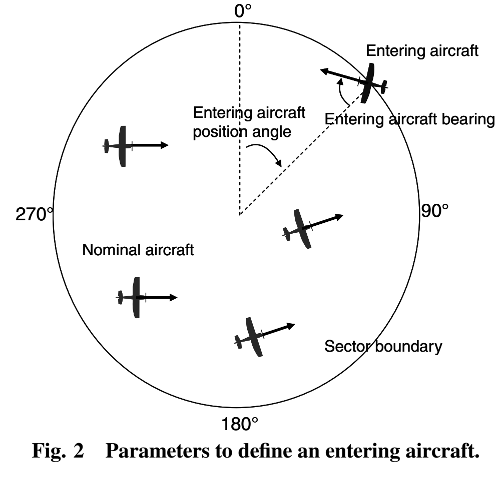


Fig. 2 Parameters to define an entering aircraft.

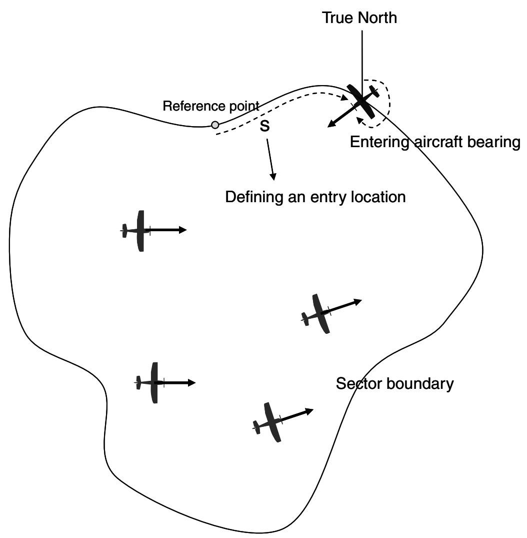


Fig. 3 Parameters to define an entering aircraft: arbitrary shape of sector boundary.

true north or some reference point within the sector, as shown in Fig. 3.

It should be noted that the method provided in the paper can accommodate other types of disturbances such as nonconforming aircraft. The flexibility provided by this method in analyzing different types of disturbances provides broad applicability.

B. Control Activity: Complexity Map

Control activity represents the degree of response required to resolve any conflict arising from a disturbance by airspace as it reacts to the disturbance. If no conflict arises from an entering aircraft and no conflict was originally present among the aircraft inside the sector, the control activity due to this particular entering aircraft is zero. This paper considers three different types of control activity. Most analyses in this paper will compute the sum of the total heading changes over all aircraft inside the sector to maintain a conflict-free situation. Another type of control activity is the sum of the heading changes due to secondary conflicts. In multiple aircraft scenarios, resolving conflicts can create new conflicts, and these newly arising conflicts are called secondary conflicts. The measure of secondary conflicts can be a good indicator of how aircraft trajectories are interrelated. The last type of control activity is the number of aircraft that undergo a heading change inside the sector. This measure might be a good indicator of the required control activity, because many small heading changes adding up to a large total heading change may require more control actions than a single large heading change.

Alternate measures are possible, including nonlinear weighting of heading changes (e.g., not including heading changes for which the amplitude is lower than a threshold considered being negligible). For more extensive airspace models (i.e., a three-dimensional airspace model), control activity might be defined by combining different types of aircraft maneuvers (e.g., the heading change, the speed change, and the altitude change) with proper weighting of each. This flexibility in choosing control activity allows tailoring the analysis to specific purposes.

The loci of these values of control activity over all possible entering-aircraft position angles and bearings are displayed as a complexity map of the immediate traffic situation. The horizontal axis of the complexity map represents the entering-aircraft position angle, and the vertical axis of the complexity map represents the entering-aircraft bearing angle.

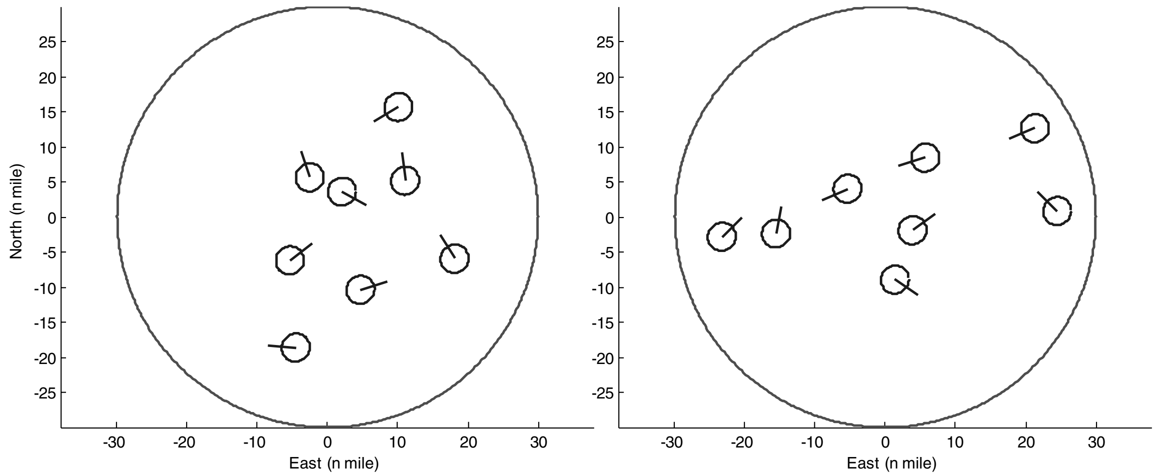
More than two parameters might be needed to define each instance of entering aircraft. For example, we may consider entering aircraft with different entry locations, bearings, and speeds. In such a case, a single two-dimensional complexity map cannot fully capture airspace response to disturbances. However, two-dimensional complexity maps can still be used by choosing two major parameters and, for each combination of these two major parameters, computing control activity over ranges of other parameters. Other types of graphical representations (e.g., a prediction profile plot) might also be used [38].

C. Numerical Examples

Two airspaces configurations in Fig. 4 (called sectors A and B, respectively) will be investigated to demonstrate our method. The large circles represent sector boundaries and the small circles represent aircraft inside the sector. The velocity vectors of the aircraft are indicated by the line segments originating from the center of the small circles. The initial configurations of traffic in both configurations are conflict-free, which means that the undisturbed propagation of all aircraft positions in time will not create any safety violation between aircraft for any time. However, this assumption can be relaxed. These traffic scenarios mimic the future free-flight environment because they have no fixed air-route structure.

1. *Complexity Map*

Using the centralized optimal conflict solver from Pallottino et al. [36], we computed complexity maps for the traffic situations in sectors A and B. These are shown in Figs. 5 and 6, respectively. These complexity maps show how each traffic situation will suffer from a disturbance (i.e., an entering aircraft) in terms of control activity.



a) Sector A **b) Sector B**

Fig. 4 Traffic situations in sectors A and B.

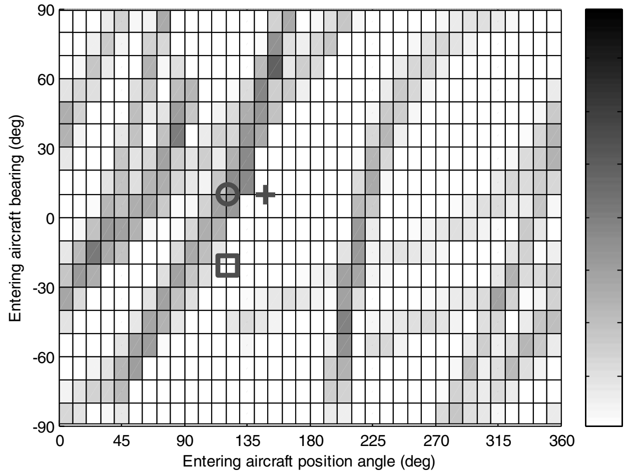


Fig. 5 Complexity map for the traffic situation in sector A. The plot indicates the sum of the total heading changes for all combinations of entering-aircraft bearing and position angles.

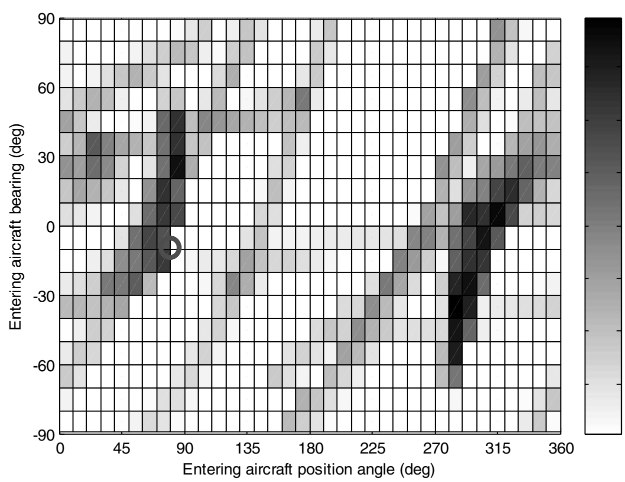


Fig. 6 Complexity map for the traffic situation in sector B. The plot indicates the sum of the total heading changes for all combinations of entering-aircraft bearing and position angles.

The complexity map for each sector shows the control activity (i.e., the total required heading changes by aircraft inside the sector) for all combinations of entering-aircraft position angle and bearings. For example, consider a particular entering aircraft for which the position angle is 120 deg and for which the bearing is 10 deg for the traffic situation in sector A. This entering aircraft is represented on the complexity map for the traffic in sector A by the symbol ○ in Fig. 5, and the required control activity resulting from this disturbance is 35 deg. However, an entering aircraft with a position angle of 150 deg and the same bearing, represented by the symbol + in Fig. 5, requires zero control activity. Similarly, an entering aircraft with position angle of 120 deg and bearing of −20 deg requires zero control activity by the current traffic in the sector A, as represented by the symbol □ in Fig. 5.

Comparing Fig. 5 with Fig. 6, we see that the traffic situation in sector B requires a nonzero control activity over a noticeably larger range of entering-aircraft position and bearing angles than with the traffic situation in sector A. Moreover, the largest control activity for the traffic situation in sector A is 49 deg, whereas this maximum value for sector B is 79 deg.

Some important information comes from the relative impact of the different entering aircraft on the airspace, as shown by the topology of the complexity map. For example, in the traffic situation in sector B, entering aircraft with position angles between 30 and 80 deg and between 280 and 350 deg will require large control activity. Based on this information, the airspace manager can decide to restrict traffic coming through this part of the sector boundary, therefore aiding TFM based on traffic complexity. As an extreme case, if an aircraft inside the sector is too close to the sector boundary to resolve a conflict with a certain range of entering aircraft, the corresponding part of the sector boundary must be closed. (This partial closure of a sector boundary will affect the complexity of the next adjacent sector, as discussed later.)

The complexity map can also identify a rapid change of control activity with respect to entry point angle and heading of the entering aircraft. For example, consider an entering aircraft for which the position angle is 80 deg and for which the bearing angle is −10 deg in the traffic situation in sector B. This entering aircraft is represented on the complexity map by the symbol ○ in Fig. 6, and the required control activity for this aircraft is zero. However, a small increase in bearing or a small decrease in position angle can result in a high required control activity.

Now, we examine how the complexity of sector B is added up by each of aircraft inside it. The traffic situation in sector B was constructed by adding one aircraft at a time from the westmost aircraft to the eastmost aircraft. The evolution of the complexity map of sector B is shown in Fig. 7. As we can see, high control activity for entering aircraft with position angles between 280 and 350 deg started growing when the second westmost aircraft was added and stopped growing after the sixth westmost aircraft was added. Likewise, another area of high control activity (which corresponds to entering aircraft with position angles between 30 and 80 deg) did not clearly appear until the last aircraft (i.e., the eastmost aircraft) was added.

We can also notice that control activity has increased over all entering aircraft by adding aircraft. However, complexity (i.e., control activity over all entering aircraft) does not always increase by adding aircraft for every conflict resolution algorithm. This is true only when a given conflict resolution algorithm can find the minimum control activity to resolve conflicts arising from an entering aircraft; the conflict resolution algorithm used in this section (i.e., the conflict solver from Pallottino et al. [36]) finds the minimum required heading changes to resolve conflicts.

2. *Complexity Map: Secondary Conflicts*

In the previous examples, initial conflicts arise from entering aircraft because the initial traffic configurations in both sectors A and B are conflict-free. However, resolving these initial conflicts created new conflicts, and the sum of required heading changes for resolving these secondary conflicts can also serve as a measure of control activity.

Using this measure of control activity, the complexity maps for the previous traffic situations in sectors A and B were computed as shown in Fig. 8. Comparing these complexity maps of secondary conflicts with the previous complexity maps in Figs. 5 and 6, a large portion of control activity in the complexity map of sector B was due to the secondary conflicts. In other words, relatively few conflicts arising from an entering aircraft create a large amount of control activity in sector B. On the other hand, for the traffic situation in sector A, entering aircraft did not create many secondary conflicts, as shown in Fig. 8a.

3. *Complexity Map: Number of Heading Changes*

In this section, we will investigate the complexity of the previous traffic situations shown in Fig. 4 with a different measure of control activity (i.e., the number of heading changes by aircraft inside the sector). This measure of control activity might be a better indicator of control activity required by the traffic controller from the disturbances, because many small heading changes adding up to a large total heading change would be more costly than a single large heading change. Complexity maps with the number of heading changes are shown for traffic situations in sectors A and B in Fig. 9.

Comparing complexity maps in Fig. 9, we see that the traffic situation in sector B requires more aircraft to change their heading angles over a noticeably larger range of entering-aircraft position and bearing angles than with the traffic situation in sector A.

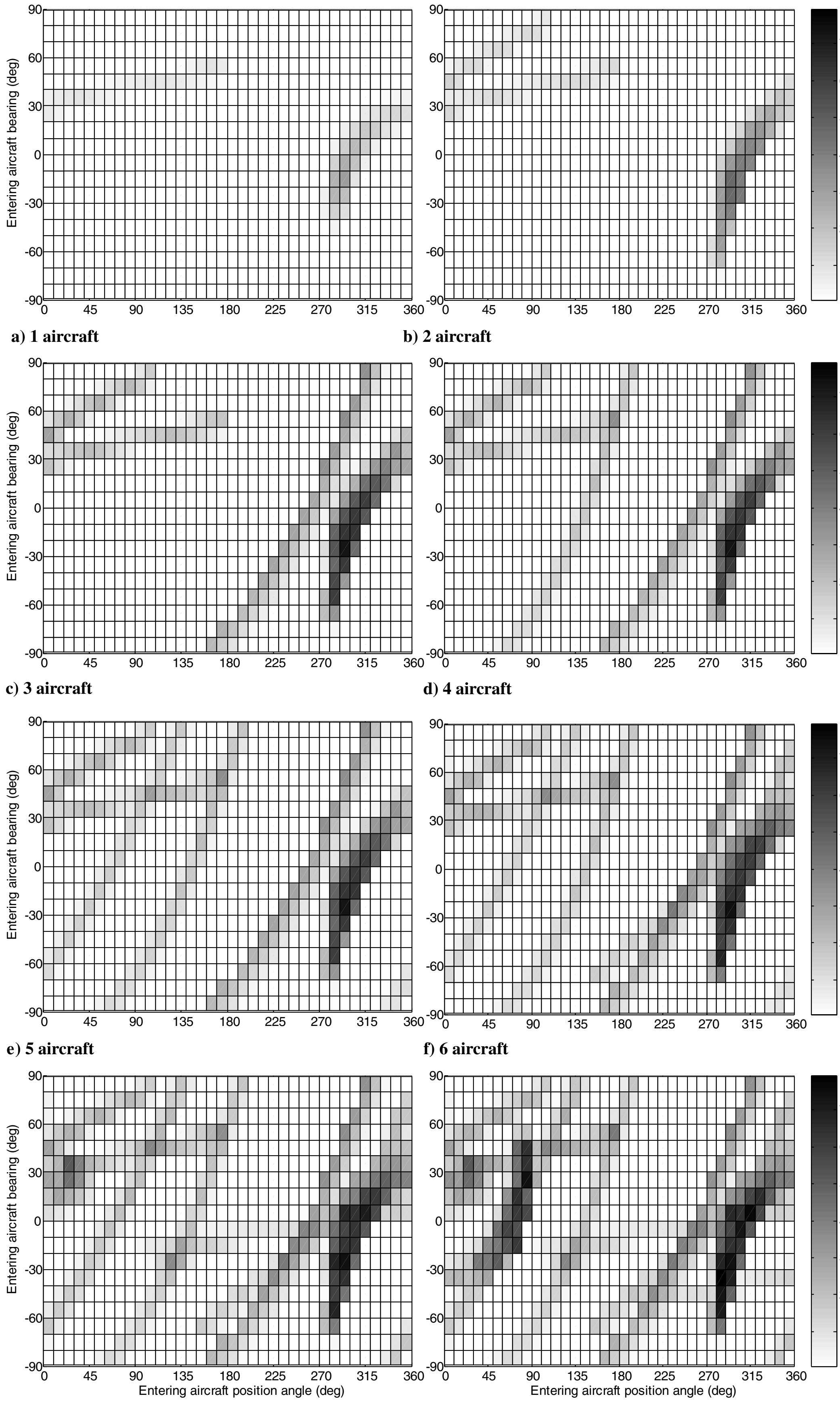


Fig. 7 Complexity added by each aircraft inside sector B.

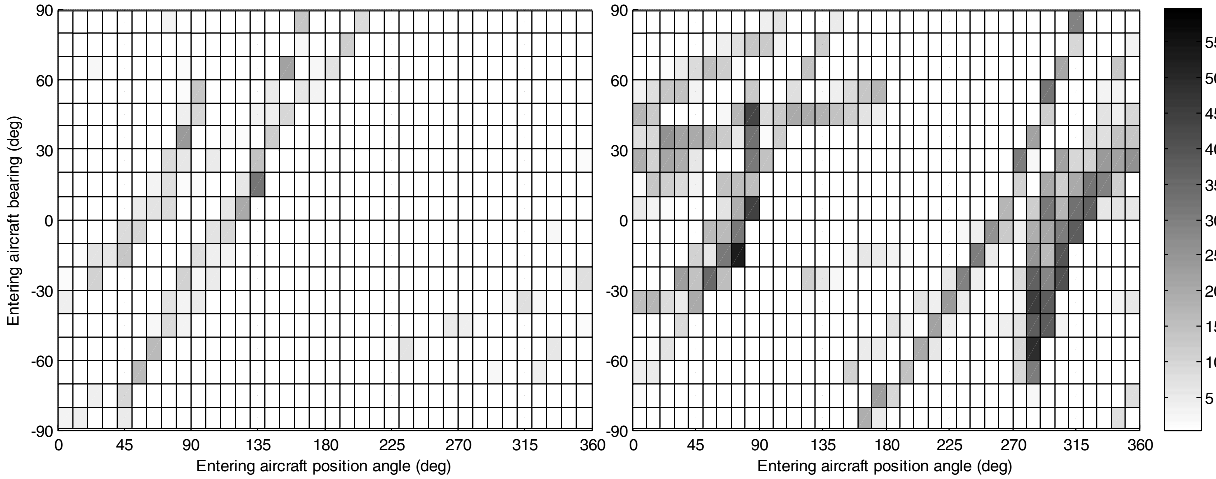


Fig. 8 Complexity maps for the traffic situations in sectors A and B. The plot indicates the sum of the total heading changes due to secondary conflicts.

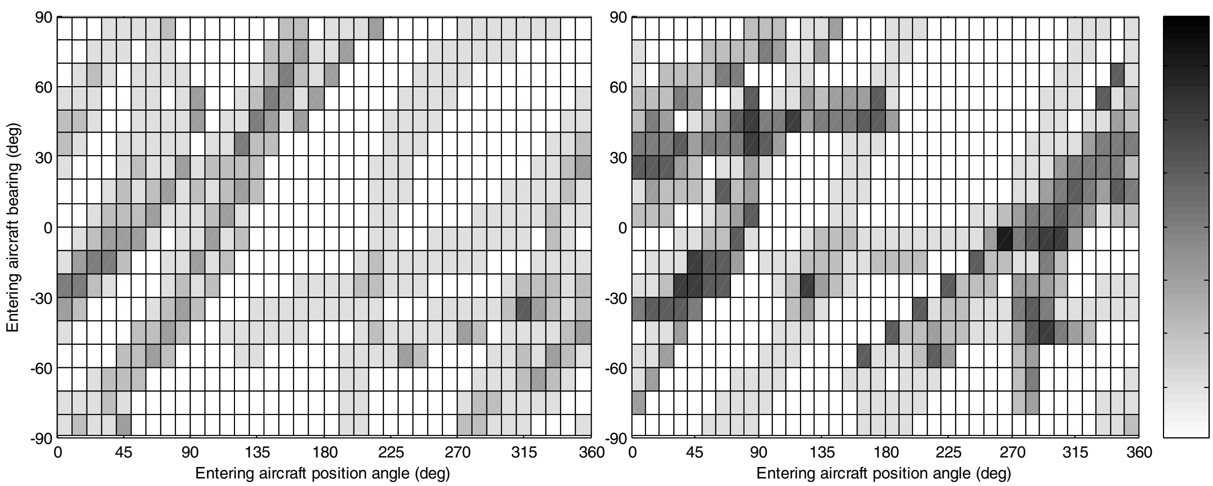


Fig. 9 Complexity maps for the traffic situations in sectors A and B. The plot indicates the number of heading changes required for all combination of entering-aircraft bearings and position angles.

We can also notice that for the traffic situation in sector B, entering aircraft with position angles between 30 and 80 deg and between 280 and 350 deg (which corresponds to the problematic part of sector boundary identified in the previous section) still require a large control activity (i.e., a large number of heading changes).

4. Complexity Map: Sequential Conflict Resolution Algorithm

This section investigates the complexity of the traffic situations in sectors A and B with the sequential conflict resolution algorithm discussed in the previous section. The sum of the total heading changes over all aircraft inside the sector was computed as a measure of control activity. Complexity maps with the sequential conflict resolution algorithm are shown for traffic situations in sectors A and B in Fig. 10.

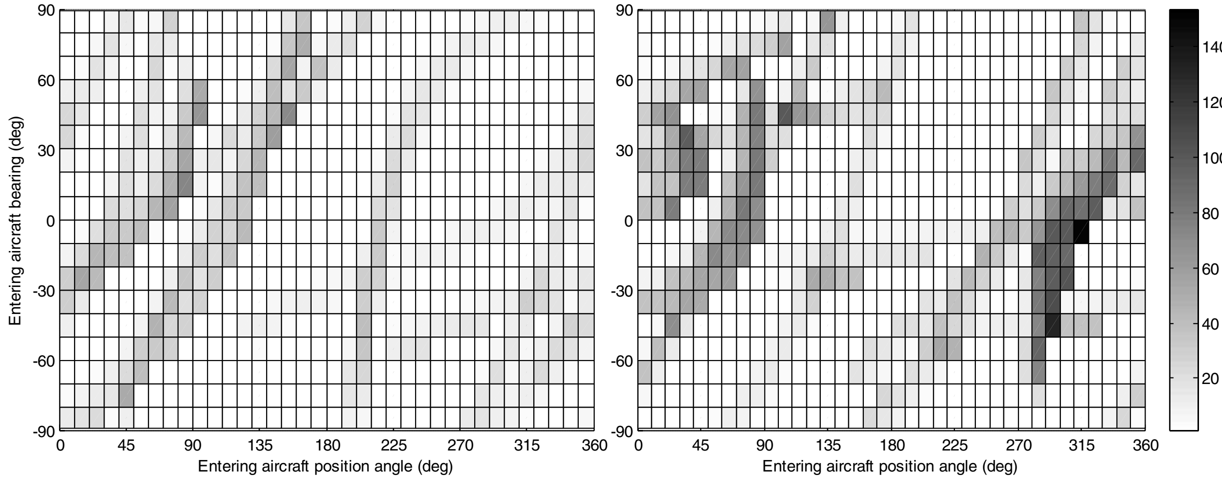
Comparing these maps with complexity maps with the mixed-integer linear programming (MILP) solver from Pallottino et al. [36], as shown in Figs. 5 and 6, the control activity increases for both traffic situations due to the suboptimality of the sequential conflict resolution algorithm. Although the affected areas for the traffic situation in sector A are very limited, the amount of control activity required within those areas increases dramatically. Correspondingly, for the traffic situation in sector B, the control activity increases dramatically for a wider range of entering-aircraft position angles and bearings, as shown in Fig. 10b. The average control activity for traffic situation in sector A is increased by 24%, and the average control activity for traffic situation in sector B is increased by 43%.

D. Scalar Measures of Airspace Complexity

The complexity map provides detailed information about the control activity required within a sector in response to disturbances. This detail may be aggregated into one of a variety of scalar measures as appropriate to the objectives of the analysis. For example, the worst-case value for required control activity may be used as an indication of the sector's sensitivity to disturbances. Alternatively, the area enclosed on a complexity map representing conditions requiring a control activity exceeding some minimum threshold provides an estimate of the range of situations requiring a response. One may also use the average of control activities. Many other methods are, of course, possible for reducing the complexity maps to scalar values, each with its own operational relevance. Table 1 shows several scalar complexity measures for the previous complexity maps.

V. Time Evolution of a Complexity Map

This section investigates how a complexity map changes over time. In the previous sections, airspace complexity was described with an instantaneous traffic situation and its responses to aircraft entering into the airspace at the moment. However, in practical



a) Sector A b) Sector B

Fig. 10 Complexity maps for the traffic situation in sectors A and B: Sequential conflict resolution algorithm. The plot indicates the sum of the total heading changes for all combinations of entering-aircraft bearing and position angles.

applications, airspace managers' decisions should be based on airspace complexity over a certain time period because the airspace requires some time to act upon airspace managers' decisions. Therefore, airspace complexity should be described over a certain period of time, and a complexity map used by the airspace managers should not change significantly for that time interval.

This paper uses a statistical approach to assess the time evolution of the complexity map. In this paper, the time evolution of a complexity map in a discrete time sequence is evaluated using the following metric. Throughout, it should be noted that a complexity map represents the control activity as a function of two parameters (i.e., the entering-aircraft position angle and the entering-aircraft bearing); these are called α and β, respectively. Therefore, a complexity map at time t_i, which represents the ith time in the discrete time sequence, can be written as $f(\alpha, \beta, t_i)$, where

$$\alpha \in \{\alpha_1 = 0\,\text{deg}, \ldots, \alpha_N = 360\,\text{deg}\}$$

and

$$\beta \in \{\beta_1 = -90\,\text{deg}, \ldots, \beta_M = 90\,\text{deg}\}$$

Now the distance between complexity maps at t_i and t_{i+1} can be evaluated by the following metric:

$$\text{difference metric} = \frac{1}{NM} \sum_{\beta=\beta_1}^{\beta=\beta_M} \sum_{\alpha=\alpha_1}^{\alpha=\alpha_N} |f(\alpha, \beta, t_{i+1}) - f(\alpha, \beta, t_i)| \tag{4}$$

As an example, consider the time evolution of the complexity map for the traffic situation in sector B over 30 s. The traffic situations in sector B at the current time and after 30 s are shown in Fig. 11 with corresponding complexity maps. Using this metric, the distance between complexity maps in Fig. 11 was computed to be 9.08 deg.

Then a range of traffic situations (with 4 aircraft within a 30 n mile radius circular airspace) was randomly created, and time evolutions of their complexity maps over 60 s were evaluated. The time evolution of a complexity map should fit within the time required to make decisions based upon it. Because of the small size of a sector (i.e., 30 n mile radius circular airspace) and the high traffic density (i.e., 14 aircraft per 10,000 n mile^2) used in our simulations, 60 s can be considered long enough to see how a complexity map changes over a time scale corresponding to traffic flow management applications. With a 99% confidence level, the confidence interval of the average value of the metric (i.e., μ) was computed to be $\mu \in (0, 12.52\,\text{deg})$. It should be noted that high values for the difference metric in the sample data usually correspond to the cases in which two conflicting aircraft are too close each other. In such a situation, resolving conflicts later in time could result in much higher control activity. Fortunately, this kind of traffic scenario must rarely happen in real situation. The result might depend on the traffic density.

VI. Applications

This section demonstrates how the proposed method of describing airspace complexity can be applied to applications of TFM and DAC. As shown later, detailed information provided by a complexity map about how airspace responds to different entering aircraft is useful to identify and resolve overly demanding traffic situations.

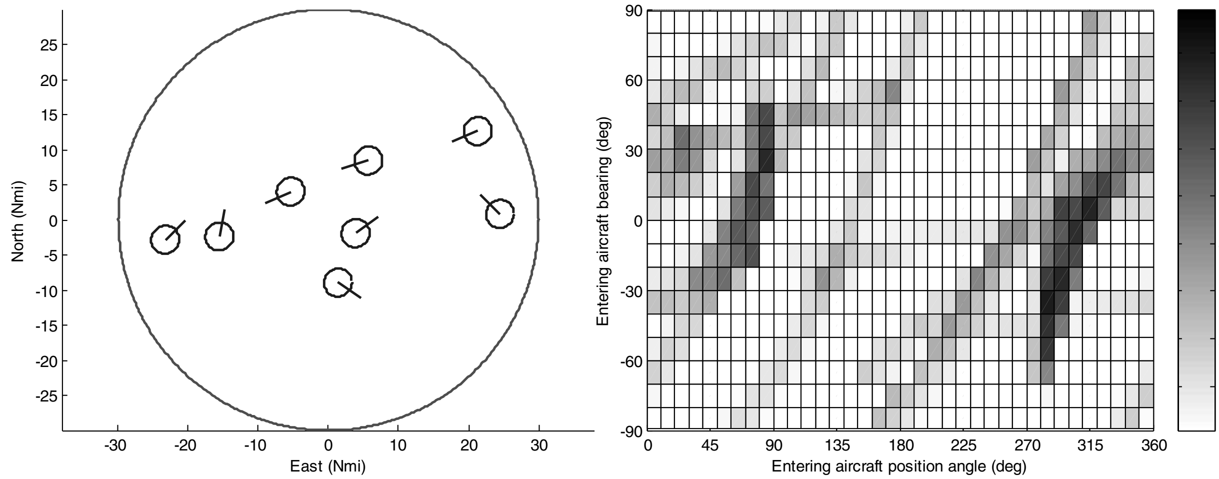
A. Airspace Restriction

Among advanced TFM concepts that have been suggested, the multisector (or multicenter) TFM have drawn attention and their benefits have been previously identified [8,39,40]. The multisector (or multicenter) TFM stresses cooperation between controllers in different sectors to best use airspace capacity. Therefore, controllers need to capture how incoming traffic from other airspace impacts their airspaces, and the method proposed in this paper can provide this capability.

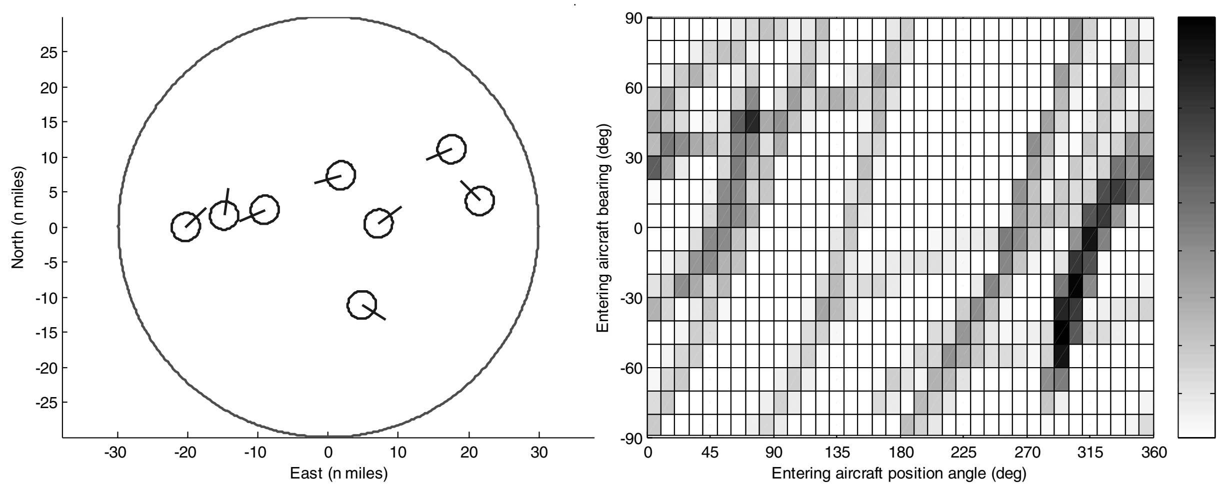
Air traffic flow managers sometimes restrict incoming traffic flows into an airspace based on its current and future traffic configurations to prevent an overly demanding traffic situation. However, because each airspace receives traffic from many others, the most effective restriction is not easily identified. The current practice is based on intuition or historical data and may result in overrestriction or underrestriction [4].

Table 1 Scalar measures of airspace complexity

	Average, deg	Max, deg
Sector A: MILP conflict resolution algorithm (Fig. 5)	6.6	49.3
Sector A: sequential conflict resolution algorithm (Fig. 10a)	8.2	73.3
Sector B: MILP conflict resolution algorithm (Fig. 6)	10.4	79.3
Sector B: sequential conflict resolution algorithm (Fig. 10b)	14.9	151.3



a) Traffic situation in sector B at current time and its complexity map



b) Traffic situation in sector B after 30 s and its complexity map

Fig. 11 Time evolution of the complexity map for sector B over 30 s.

Because a complexity map provides detailed information about airspace response to any entering aircraft, air traffic flow managers can easily identify the problematic part of the sector boundary that should be closed. For example, consider the traffic situation in sector B, as shown in Fig. 4b. Based on its complexity map, as shown in Fig. 6, traffic flow managers can easily identify that entering aircraft with position angles between 30 and 80 deg and between 280 and 350 deg will require large control activity by aircraft in sector B.

Assume that, based on this information, air traffic flow managers decide to restrict the traffic flows into sector B through the sector boundary corresponding to aircraft entering with position angles between 280 and 350 deg. Such restrictions on the incoming traffic flows impact the complexity of the upstream airspace. Assume that sector A is one of the adjacent sectors of sector B, so that a part of the sector A's boundary is closed, as shown in Fig. 12. Therefore, aircraft inside the sector A are not allowed to exit through that part of the sector boundary, and the complexity of sector A should be described with this boundary constraint. Note that the traffic situation in sector A is not currently conflicting with this boundary closure constraint because their routes should be already adjusted to meet this constraint. However, the complexity of this traffic will be affected because its response to a disturbance is limited by this boundary closure. The complexity maps of sector A with and without this boundary closure are shown in Fig. 13. Comparing these complexity maps, as shown in Fig. 13, the control activity increases, but the affected areas are limited in range. This information helps air traffic flow managers to identify how their decision will affect other

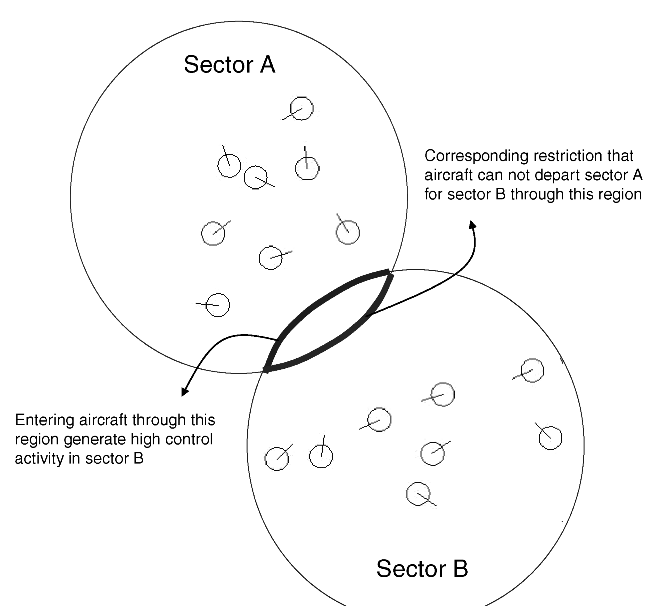
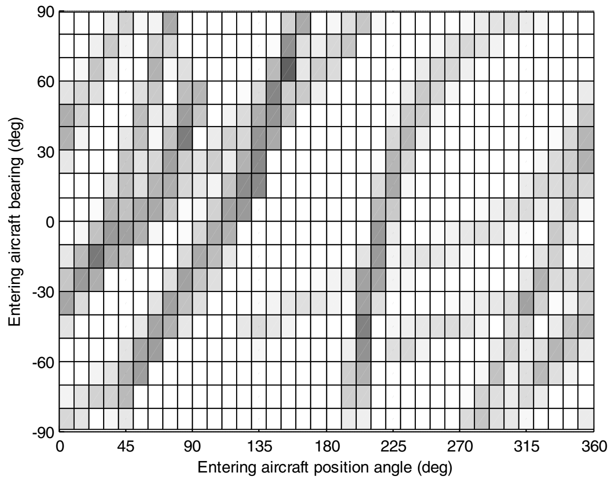
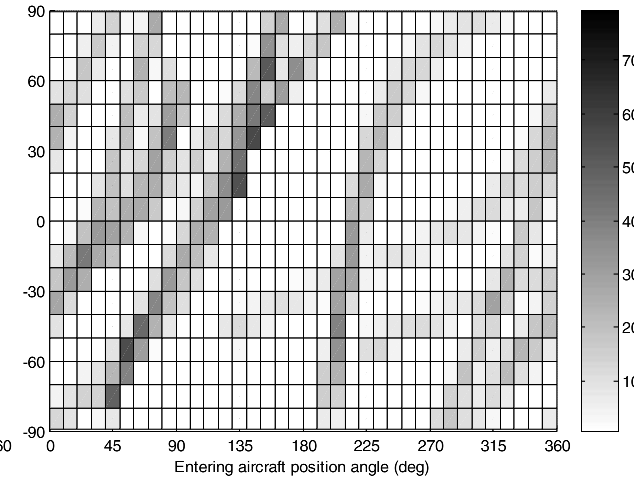


Fig. 12 Partial closure of the sector boundary.



a) Sector A: no sector boundary closure



b) Sector A: partially closed boundary

Fig. 13 Complexity maps for the traffic situation in sector A: with and without partially closed boundary.

airspaces and to decide whether or not to partially close this segment of the boundary between sectors A and B.

This example illustrates the capabilities of the proposed method by which air traffic flow managers can identify overly demanding traffic situations and determine the effective flow restrictions. The proposed method of a complexity map also allows air traffic managers to assess how their sector-closure decisions will affect the complexity of the adjacent airspace. Detailed information about how a particular change in one airspace affects the complexity of other airspace is necessary for the cooperation between multiple airspace, and the proposed method can provide this capability.

Although this section focused on the traffic flow management, quantifying the degree of increased complexity caused by partial closure of a sector's boundary is also necessary for decision-makers in the dynamic airspace configuration. For example, in the dynamic airspace configuration, large volumes of airspace are regularly restricted by airspace managers for other reasons such as military or space launch operations [41], and airspace managers need to assess how their sector-closing decisions will affect the complexity of other sectors.

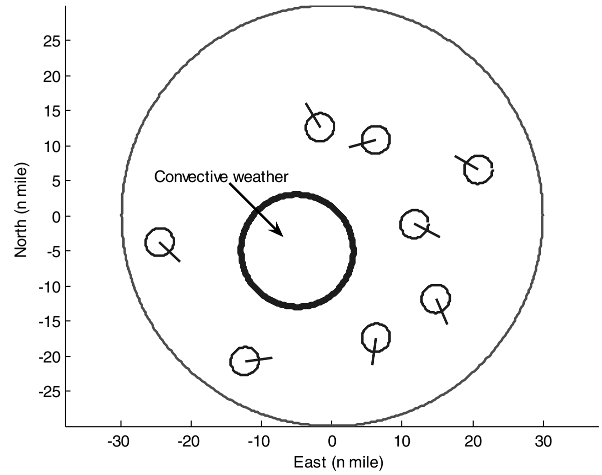


Fig. 14 Traffic situation in sector C.

B. Traffic Flow Management in Convective Weather

Convective weather is one of the leading causes of delays in the NAS [3], and it is important to assess how convective weather affects airspace complexity. In this section, it will be shown that the proposed method can incorporate convective weather and allow air traffic flow managers to identify an effective way to manage traffic flows in the presence of convective weather.

For example, consider the airspace called sector C in Fig. 14; its complexity map is shown in Fig. 15a. Now assume that convective weather exists in sector C. Convective weather is modeled as a circular closed region in the airspace, as shown in Fig. 14, and the traffic inside the sector is not allowed to go through it. We assume that the current traffic in sector C is not conflicting with this convective weather constraint, but the complexity of this traffic will be affected because its response to a disturbance is limited by this constraint.

The complexity map for sector C with this convective weather is shown in Fig. 15b. Comparing the complexity maps with and without this convective weather in Fig. 15, the affected areas by this convective weather are very limited. However, the control activity for entering aircraft with position angles between 230 and 250 deg significantly increased over a wide range of entering-aircraft bearing angles.

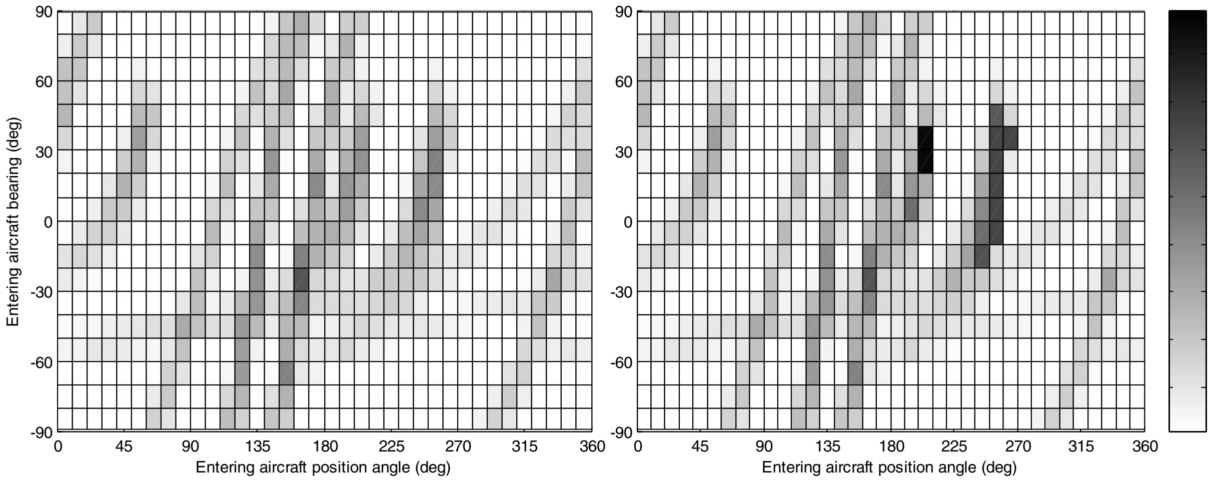
Furthermore, based on the complexity maps of sector C and its adjacent sectors, air traffic flow managers can identify the most efficient way to reroute traffic flows, because a complexity map provides information about how each sector can suffer from incoming aircraft.

C. Inherent Complexity of the Air-Route Structure

So far, airspace complexity of a given traffic situation was described. However, the study of airspace complexity can also be applied to evaluate different airspace configurations. This section demonstrates how the proposed method can be applied to evaluate the complexity of a given air-route structure. The inherent complexity of a given air-route structure is described by its complexity over a wide range of traffic situations.

As an example, consider the air-route structure with two air routes, as shown in Fig. 16. For this air-route structure, complexity maps of a wide range of traffic situations were computed and then their average was computed as an inherent complexity map of the air-route structure, as shown in Fig. 17. For each simulation, three aircraft on route 1 and two aircraft on route 2 were randomly created.

This inherent complexity map has areas with high control activity around the exit location of both air routes. In other words, entering aircraft with position angles between 15 and 45 deg and between 135 and 165 deg will require significant control activity. Using this information, air traffic controllers can focus on a specific range of the sector boundary instead of equally distributing their attention around the boundary.



a) Sector C without convective weather **b) Sector C with convective weather**

Fig. 15 Complexity maps for the traffic situation in sector C: with and without convective weather.

A given air-route structure can be compared with other structures by assessing its inherent complexity over a wide range of traffic situations. As an example, we are currently comparing a set of air-route structures (each had two air routes with different crossing angles) using statistical analysis.

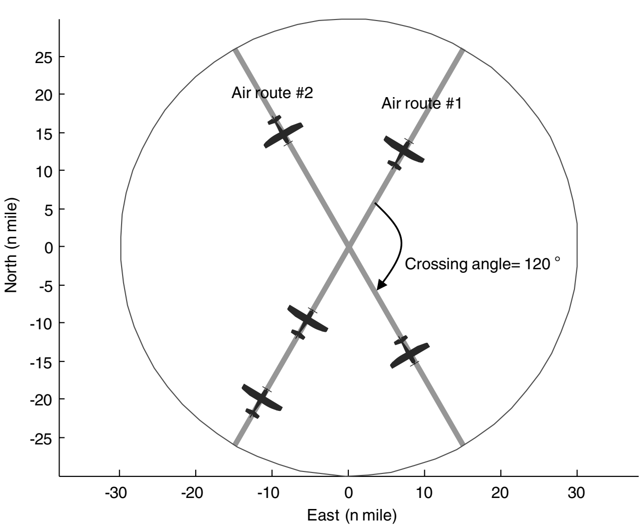


Fig. 16 Air-route structure: two routes with crossing angles of 120 deg.

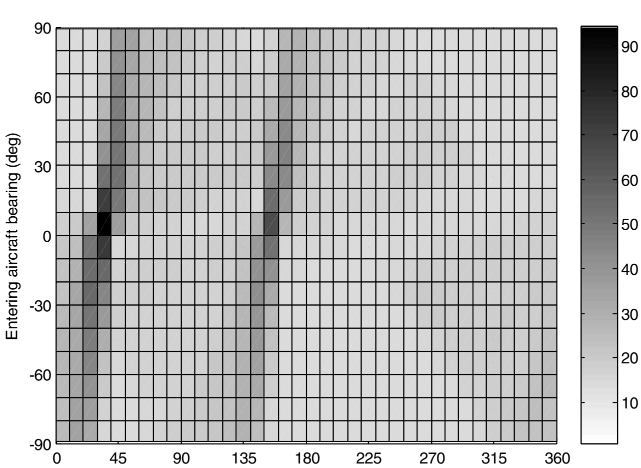


Fig. 17 Inherent complexity map for the air-route structure in Fig. 16.

VII. Conclusions

Traffic flow management and dynamic airspace configuration require efficient use of the National Airspace System in the face of imbalance between air traffic demand and airspace capacity. To that end, airspace complexity stands as a fundamental research problem, and this paper proposed a new notion of airspace complexity with the viewpoint that an airspace is a closed-loop controlled system responding to various disturbances. Because the difficulty of dealing with a given traffic situation depends on the disturbance, a single scalar metric cannot properly describe the airspace complexity. Instead, a complexity map provides detailed information about how the traffic responds to disturbances such as an aircraft entering into the sector, and we showed that this information is relevant for traffic flow management and dynamic airspace configuration.

A complexity map can also provide information about how environmental factors such as convective weather will affect airspace complexity. The proposed method of describing airspace complexity can also be applied to evaluating different air-route structures. The inherent complexity of a given air-route structure is described by assessing its complexity over a wide range of traffic situations. The time evolution of a complexity map was also investigated. The time evolution of a complexity map is of interest because, in a real application, airspace complexity should be described over a certain period time rather than at an instant in time.

Future work should extend the models for each component in the air traffic control system, while remaining computationally tractable. The two-dimensional airspace model might need to be extended to a three-dimensional model, and different traffic control methods should be identified. Working with the operational community, we might need to identify the proper measures of control activity to represent the workload experienced by the system from the disturbances.

The paper focused on describing how an airspace will suffer from disturbances (i.e., entering aircraft). Future investigation should examine how airspace complexity changes after a new aircraft enters into the sector. Therefore, traffic flow managers decide whether or not to accept an entering aircraft by considering not only how an airspace will suffer from the entrance of an aircraft, but also how the complexity map of an airspace will change after accepting an entering aircraft.

We also need to identify and characterize other types of disturbances. For example, nonconforming aircraft can be considered as a disturbance in airspace, and traffic controllers can identify the aircraft for which the nonconformance create the most serious problem in the system by evaluating the response of the airspace to different nonconformance events. The choice of disturbance should be closely tied with its practical application, and the flexibility provided by this method in analyzing a range of disturbances provides broad applicability.

With appropriate models, we can develop computational tools that can be used to evaluate the complexity of a given traffic situation. By exhaustive analyses over a large range of traffic situations, this research can provide a unified approach to evaluate any proposed automated air traffic control system. For example, the proposed method can be applied to evaluating any type of airborne separation-assistance systems in terms of how they can handle a range of traffic situations in the face of different types of disturbances.

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